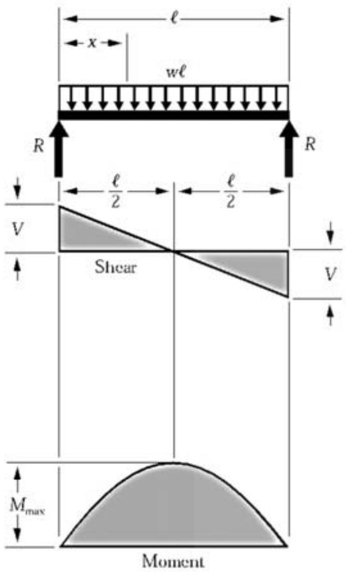
Chapter (7): Useful Formulas

7.1 Beam Design Formulas with Shear and Moment Diagrams

Figure 1 Simple Beam – Uniformly Distributed Load



$R = V \quad \ldots \quad = \frac{w\ell}{2}$

$V_x \quad \ldots \quad = w\left(\frac{\ell}{2} - x\right)$

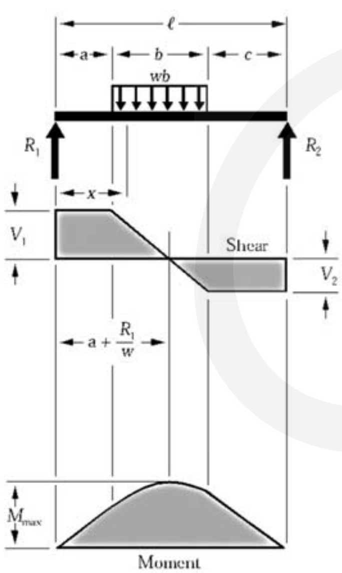
M_{max} (at center) $\ldots = \frac{w\ell^2}{8}$

$M_x \quad \ldots \quad = \frac{wx}{2}(\ell - x)$

Δ_{max} (at center) $\ldots = \frac{5w\ell^4}{384\,EI}$

$\Delta_x \quad \ldots \quad = \frac{wx}{24\,EI}(\ell^3 - 2\ell x^2 + x^3)$

Figure 2 Simple Beam – Uniform Load Partially Distributed



$R_1 = V_1$ (max when $a < c$) $\ldots = \frac{wb}{2\ell}(2c + b)$

$R_2 = V_2$ (max when $a > c$) $\ldots = \frac{wb}{2\ell}(2a + b)$

V_x (when $x > a$ and $< (a + b)$) $\ldots = R_1 - w(x - a)$

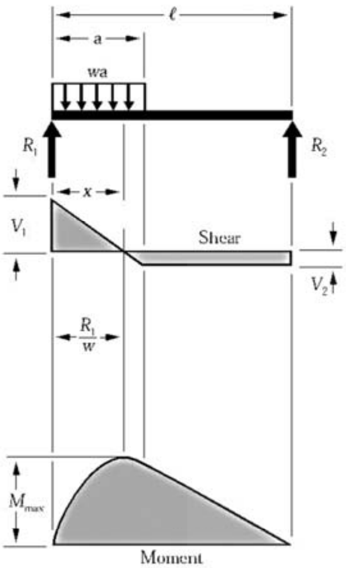
$M_{max}\left(\text{at } x = a + \frac{R_1}{w}\right) \ldots = R_1\left(a + \frac{R_1}{2w}\right)$

M_x (when $x < a$) $\ldots = R_1 x$

M_x (when $x > a$ and $< (a + b)$) $\ldots = R_1 x - \frac{w}{2}(x - a)^2$

M_x (when $x > (a + b)$) $\ldots = R_2(\ell - x)$

Figure 3 Simple Beam – Uniform Load Partially Distributed at One End



$$R_1 = V_1 \quad \dots\dots\dots\dots = \frac{wa}{2\ell}(2\ell - a)$$

$$R_2 = V_2 \quad \dots\dots\dots\dots = \frac{wa^2}{2\ell}$$

$$V_x \text{ (when } x < a) \quad \dots\dots = R_1 - wx$$

$$M_{max}\left(\text{at } x = \frac{R_1}{w}\right) \quad \dots\dots = \frac{R_1^{\,2}}{2w}$$

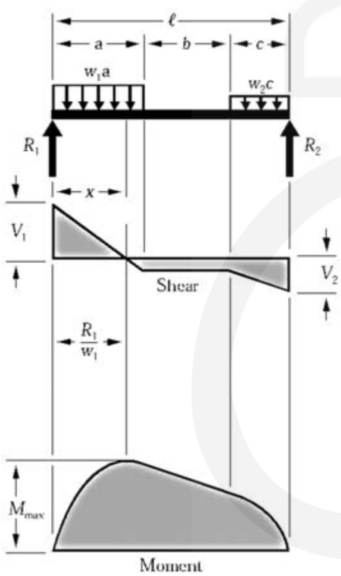
$$M_x \text{ (when } x < a) \quad \dots\dots = R_1 x - \frac{wx^2}{2}$$

$$M_x \text{ (when } x > a) \quad \dots\dots = R_2(\ell - x)$$

$$\Delta_x \text{ (when } x < a) \quad \dots\dots = \frac{wx}{24\,EI\ell}\left(a^2(2\ell - a)^2 - 2ax^2(2\ell - a) + \ell x^3\right)$$

$$\Delta_x \text{ (when } x > a) \quad \dots\dots = \frac{wa^2(\ell - x)}{24\,EI\ell}(4x\ell - 2x^2 - a^2)$$

Figure 4 Simple Beam – Uniform Load Partially Distributed at Each End



$$R_1 = V_1 \quad \dots\dots\dots\dots\dots\dots = \frac{w_1 a(2\ell - a) + w_2 c^2}{2\ell}$$

$$R_2 = V_2 \quad \dots\dots\dots\dots\dots\dots = \frac{w_2 c(2\ell - c) + w_1 a^2}{2\ell}$$

$$V_x \text{ (when } x < a) \quad \dots\dots\dots\dots = R_1 - w_1 x$$

$$V_x \left(\text{when } x > a \text{ and } < (a + b)\right) \quad \dots = R_1 - w_1 a$$

$$V_x \left(\text{when } x > (a + b)\right) \quad \dots\dots = R_2 - w_2(\ell - x)$$

$$M_{max}\left(\text{at } x = \frac{R_1}{w_1} \text{ when } R_1 < w_1 a\right) \dots = \frac{R_1^{\,2}}{2w_1}$$

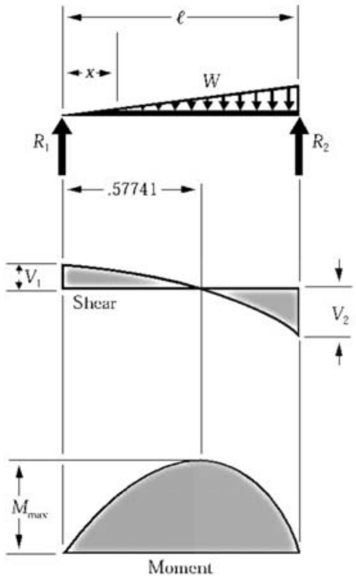
$$M_{max}\left(\text{at } x = \ell - \frac{R_2}{w_2} \text{ when } R_2 < w_2 c\right) = \frac{R_2^{\,2}}{2w_2}$$

$$M_x \text{ (when } x < a) \quad \dots\dots\dots\dots = R_1 x - \frac{w_1 x^2}{2}$$

$$M_x \left(\text{when } x > a \text{ and } < (a + b)\right) \dots = R_1 x - \frac{w_1 a}{2}(2x - a)$$

$$M_x \left(\text{when } x > (a + b)\right) \quad \dots\dots = R_2(\ell - x) - \frac{w_2(\ell - x)^2}{2}$$

Figure 5 Simple Beam – Load Increasing Uniformly to One End



$$R_1 = V_1 \;\ldots\ldots\ldots\ldots\; = \frac{W}{3}$$

$$R_2 = V_2 \;\ldots\ldots\ldots\ldots\; = \frac{2W}{3}$$

$$V_x \;\ldots\ldots\ldots\ldots\; = \frac{W}{3} - \frac{Wx^2}{\ell^2}$$

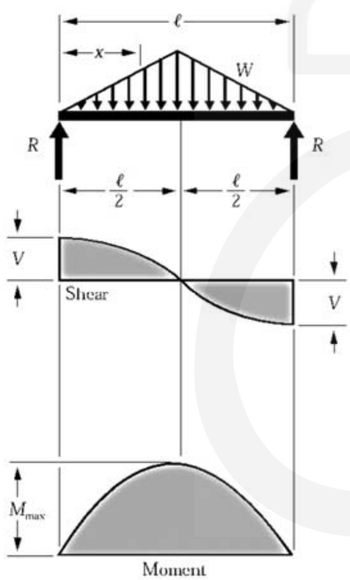
$$M_{max}\left(\text{at } x = \frac{\ell}{\sqrt{3}} = .5774\ell\right) \;\ldots\; = \frac{2W\ell}{9\sqrt{3}} = .1283W\ell$$

$$M_x \;\ldots\ldots\ldots\ldots\; = \frac{Wx}{3\ell^2}(\ell^2 - x^2)$$

$$\Delta_{max}\left(\text{at } x = \ell\sqrt{1-\sqrt{\frac{8}{15}}} = .5193\ell\right) \;.\; = .01304\frac{W\ell^3}{EI}$$

$$\Delta_x \;\ldots\ldots\ldots\ldots\; = \frac{Wx}{180EI\ell^2}(3x^4 - 10\ell^2x^2 + 7\ell^4)$$

Figure 6 Simple Beam – Load Increasing Uniformly to Center



$$R = V \;\ldots\ldots\ldots\ldots\; = \frac{W}{2}$$

$$V_x\left(\text{when } x < \frac{\ell}{2}\right) \;\ldots\ldots\; = \frac{W}{2\ell^2}(\ell^2 - 4x^2)$$

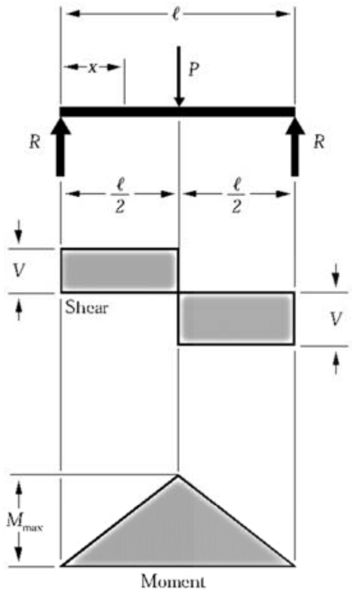
$$M_{max}\ (\text{at center}) \;\ldots\ldots\; = \frac{W\ell}{6}$$

$$M_x\left(\text{when } x < \frac{\ell}{2}\right) \;\ldots\ldots\; = Wx\left(\frac{1}{2} - \frac{2x^2}{3\ell^2}\right)$$

$$\Delta_{max}\ (\text{at center}) \;\ldots\ldots\; = \frac{W\ell^3}{60EI}$$

$$\Delta_x \;\ldots\ldots\ldots\ldots\; = \frac{Wx}{480EI\ell^2}(5\ell^2 - 4x^2)^2$$

Figure 7 Simple Beam – Concentrated Load at Center



$$R = V \quad \ldots\ldots\ldots\ldots \quad = \frac{P}{2}$$

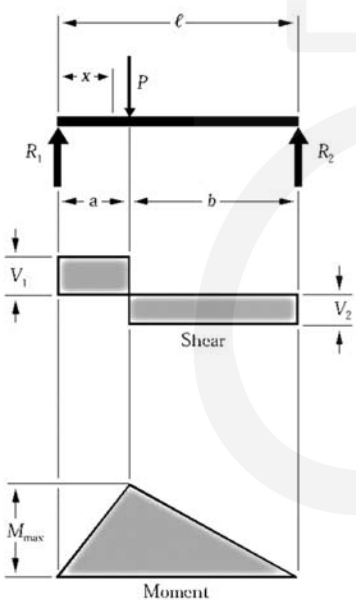
$$M_{max} \text{ (at point of load)} \quad \ldots\ldots \quad = \frac{P\ell}{4}$$

$$M_x \left(\text{when } x < \frac{\ell}{2}\right) \quad \ldots\ldots \quad = \frac{Px}{2}$$

$$\Delta_{max} \text{ (at point of load)} \quad \ldots\ldots \quad = \frac{P\ell^3}{48EI}$$

$$\Delta_x \left(\text{when } x < \frac{\ell}{2}\right) \quad \ldots\ldots \quad = \frac{Px}{48EI}(3\ell^2 - 4x^2)$$

Figure 8 Simple Beam – Concentrated Load at Any Point



$$R_1 = V_1 \text{ (max when } a < b) \quad \ldots\ldots \quad = \frac{Pb}{\ell}$$

$$R_2 = V_2 \text{ (max when } a > b) \quad \ldots\ldots \quad = \frac{Pa}{\ell}$$

$$M_{max} \text{ (at point of load)} \quad \ldots\ldots \quad = \frac{Pab}{\ell}$$

$$M_x \text{ (when } x < b) \quad \ldots\ldots \quad = \frac{Pbx}{\ell}$$

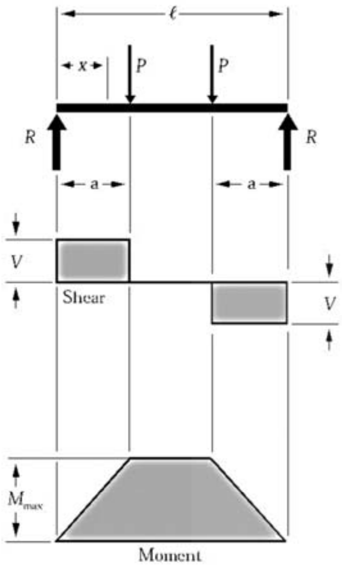
$$\Delta_{max} \left(\text{at } x = \sqrt{\frac{a(a+2b)}{3}} \text{ when } a > b\right) \quad . \quad = \frac{Pab(a+2b)\sqrt{3a(a+2b)}}{27EI\ell}$$

$$\Delta_a \text{ (at point of load)} \quad \ldots\ldots \quad = \frac{Pa^2b^2}{3EI\ell}$$

$$\Delta_x \text{ (when } x < a) \quad \ldots\ldots \quad = \frac{Pbx}{6EI\ell}(\ell^2 - b^2 - x^2)$$

$$\Delta_x \text{ (when } x > a) \quad \ldots\ldots \quad = \frac{Pa(\ell - x)}{6EI\ell}(2\ell x - x^2 - a^2)$$

Figure 9 Simple Beam – Two Equal Concentrated Loads Symmetrically Placed



$R = V$ . . . . $= P$

M_{max} (between loads) . . . . $= Pa$

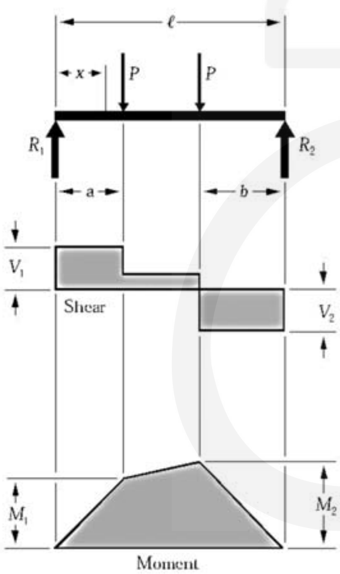
M_x (when $x < a$) . . . . $= Px$

Δ_{max} (at center) . . . . $= \frac{Pa}{24EI}(3\ell^2 - 4a^2)$

Δ_x (when $x < a$) . . . . $= \frac{Px}{6EI}(3\ell a - 3a^2 - x^2)$

Δ_x (when $x > a$ and $< (\ell - a)$) . . . $= \frac{Pa}{6EI}(3\ell x - 3x^2 - a^2)$

Figure 10 Simple Beam – Two Equal Concentrated Loads Unsymmetrically Placed



$R_1 = V_1$ (max when $a < b$) . . . . . $= \frac{P}{\ell}(\ell - a + b)$

$R_2 = V_2$ (max when $a > b$) . . . . . $= \frac{P}{\ell}(\ell - b + a)$

V_x (when $x > a$ and $< (\ell - b)$) . . . $= \frac{P}{\ell}(b - a)$

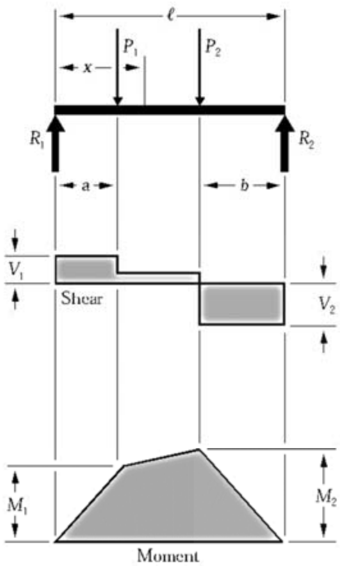
M_1 (max when $a > b$) . . . . $= R_1 a$

M_2 (max when $a < b$) . . . . $= R_2 b$

M_x (when $x < a$) . . . . $= R_1 x$

M_x (when $x > a$ and $< (\ell - b)$) . . . $= R_1 x - P(x - a)$

Figure 11 Simple Beam – Two Unequal Concentrated Loads Unsymmetrically Placed



$R_1 = V_1$ . . . . $= \dfrac{P_1(\ell - a) + P_2 b}{\ell}$

$R_2 = V_2$ . . . . $= \dfrac{P_1 a + P_2(\ell - b)}{\ell}$

V_x $\left(\text{when } x > a \text{ and } < (\ell - b)\right)$ . . . $= R_1 - P_1$

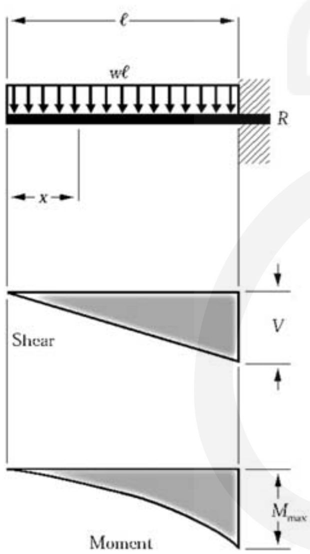
M_1 (max when $R_1 < P_1$) . . . . $= R_1 a$

M_2 (max when $R_2 < P_2$) . . . . $= R_2 b$

M_x (when $x < a$) . . . . $= R_1 x$

M_x $\left(\text{when } x > a \text{ and } < (\ell - b)\right)$ . . . $= R_1 x - P_1(x - a)$

Figure 12 Cantilever Beam – Uniformly Distributed Load



$R = V$ . . . . $= w\ell$

V_x . . . . $= wx$

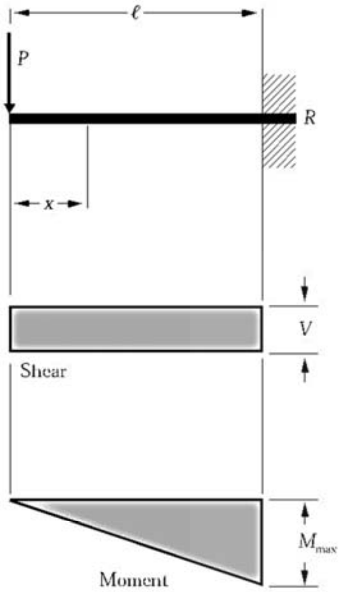
M_{max} (at fixed end) . . . . $= \dfrac{w\ell^2}{2}$

M_x . . . . $= \dfrac{wx^2}{2}$

Δ_{max} (at free end) . . . . $= \dfrac{w\ell^4}{8EI}$

Δ_x . . . . $= \dfrac{w}{24EI}(x^4 - 4\ell^3 x + 3\ell^4)$

Figure 13 **Cantilever Beam – Concentrated Load at Free End**



$R = V$ . . . . $= P$

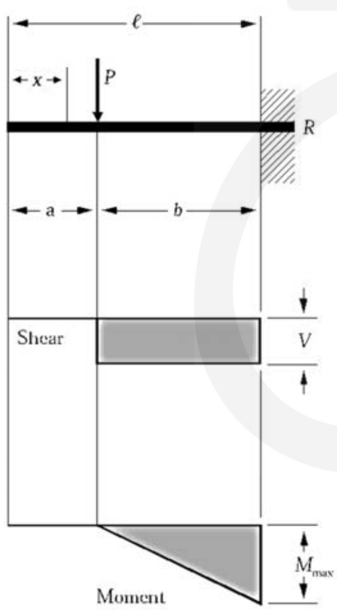
M_{max} (at fixed end) . . . . $= P\ell$

M_x . . . . $= Px$

Δ_{max} (at free end) . . . . $= \dfrac{P\ell^3}{3EI}$

Δ_x . . . . $= \dfrac{P}{6EI}(2\ell^3 - 3\ell^2 x + x^3)$

Figure 14 **Cantilever Beam – Concentrated Load at Any Point**



$R = V$ . . . . $= P$

M_{max} (at fixed end) . . . . $= Pb$

M_x (when $x > a$) . . . . $= P(x - a)$

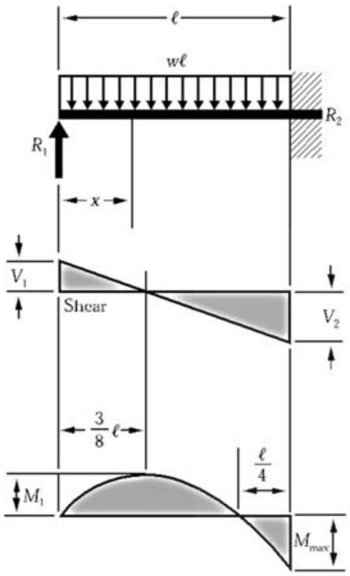
Δ_{max} (at free end) . . . . $= \dfrac{Pb^2}{6EI}(3\ell - b)$

Δ_a (at point of load) . . . . $= \dfrac{Pb^3}{3EI}$

Δ_x (when $x < a$) . . . . $= \dfrac{Pb^2}{6EI}(3\ell - 3x - b)$

Δ_x (when $x > a$) . . . . $= \dfrac{P(\ell - x)^2}{6EI}(3b - \ell + x)$

Figure 15 Beam Fixed at One End, Supported at Other – Uniformly Distributed Load



$R_1 = V_1 \quad \dots \quad = \dfrac{3w\ell}{8}$

$R_2 = V_2 \quad \dots \quad = \dfrac{5w\ell}{8}$

$V_x \quad \dots \quad = R_1 - wx$

$M_{max} \quad \dots \quad = \dfrac{w\ell^2}{8}$

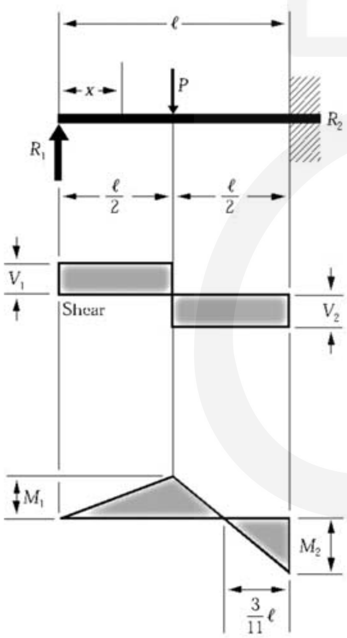
$M_1 \left(\text{at } x = \dfrac{3}{8}\ell\right) \quad \dots \quad = \dfrac{9}{128} w\ell^2$

$M_x \quad \dots \quad = R_1 x - \dfrac{wx^2}{2}$

$\Delta_{max} \left(\text{at } x = \dfrac{\ell}{16}(1 + \sqrt{33}) = .4215\,\ell\right) \quad \dots \quad = \dfrac{w\ell^4}{185EI}$

$\Delta_x \quad \dots \quad = \dfrac{wx}{48EI}(\ell^3 - 3\ell x^2 + 2x^3)$

Figure 16 Beam Fixed at One End, Supported at Other – Concentrated Load at Center



$R_1 = V_1 \quad \dots \quad = \dfrac{5P}{16}$

$R_2 = V_2 \quad \dots \quad = \dfrac{11P}{16}$

M_{max} (at fixed end) $\quad \dots \quad = \dfrac{3P\ell}{16}$

M_1 (at point of load) $\quad \dots \quad = \dfrac{5P\ell}{32}$

$M_x \left(\text{when } x < \dfrac{\ell}{2}\right) \quad \dots \quad = \dfrac{5Px}{16}$

$M_x \left(\text{when } x > \dfrac{\ell}{2}\right) \quad \dots \quad = P\left(\dfrac{\ell}{2} - \dfrac{11x}{16}\right)$

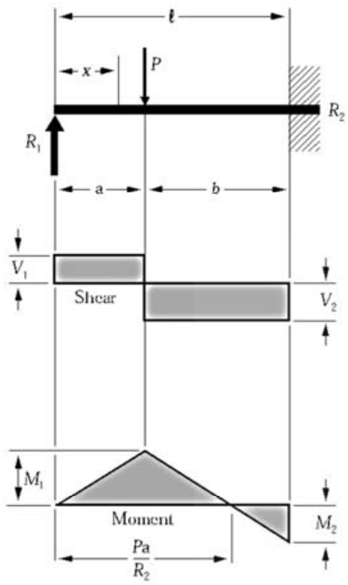
$\Delta_{max} \left(\text{at } x = \ell\sqrt{\dfrac{1}{5}} = .4472\ell\right) \quad \dots \quad = \dfrac{P\ell^3}{48EI\sqrt{5}} = .009317\dfrac{P\ell^3}{EI}$

Δ_x (at point of load) $\quad \dots \quad = \dfrac{7P\ell^3}{768EI}$

$\Delta_x \left(\text{when } x < \dfrac{\ell}{2}\right) \quad \dots \quad = \dfrac{Px}{96EI}(3\ell^2 - 5x^2)$

$\Delta_x \left(\text{when } x > \dfrac{\ell}{2}\right) \quad \dots \quad = \dfrac{P}{96EI}(x - \ell)^2(11x - 2\ell)$

Figure 17 Beam Fixed at One End, Supported at Other – Concentrated Load at Any Point



$R_1 = V_1 \ldots = \dfrac{Pb^2}{2\ell^3}(a + 2\ell)$

$R_2 = V_2 \ldots = \dfrac{Pa}{2\ell^3}(3\ell^2 - a^2)$

M_1 (at point of load) $\ldots = R_1 a$

M_2 (at fixed end) $\ldots = \dfrac{Pab}{2\ell^2}(a + \ell)$

M_x (when $x < a$) $\ldots = R_1 x$

M_x (when $x > a$) $\ldots = R_1 x - P(x - a)$

$\Delta_{max}\left(\text{when } a < .414\ell \text{ at } x = \ell\dfrac{\ell^2 + a^2}{3\ell^2 - a^2}\right) = \dfrac{Pa}{3EI}\dfrac{(\ell^2 - a^2)^3}{(3\ell^2 - a^2)^2}$

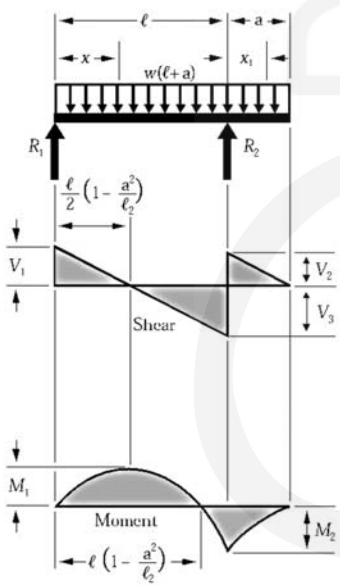
$\Delta_{max}\left(\text{when } a > .414\ell \text{ at } x = \ell\sqrt{\dfrac{a}{2\ell + a}}\right) = \dfrac{Pab^2}{6EI}\sqrt{\dfrac{a}{2\ell + a}}$

Δ_a (at point of load) $\ldots = \dfrac{Pa^2b^3}{12EI\ell^3}(3\ell + a)$

Δ_x (when $x < a$) $\ldots = \dfrac{Pb^2x}{12EI\ell^3}(3a\ell^2 - 2\ell x^2 - ax^2)$

Δ_x (when $x > a$) $\ldots = \dfrac{Pa}{12EI\ell^3}(\ell - x)^2(3\ell^2 x - a^2 x - 2a^2\ell)$

Figure 18 Beam Overhanging One Support – Uniformly Distributed Load



$R_1 = V_1 \ldots = \dfrac{w}{2\ell}(\ell^2 - a^2)$

$R_2 = V_2 + V_3 \ldots = \dfrac{w}{2\ell}(\ell + a)^2$

$V_2 \ldots = wa$

$V_3 \ldots = \dfrac{w}{2\ell}(\ell^2 + a^2)$

V_x (between supports) $\ldots = R_1 - wx$

V_{x_1} (for overhang) $\ldots = w(a - x_1)$

$M_1\left(\text{at } x = \dfrac{\ell}{2}\left[1 - \dfrac{a^2}{\ell^2}\right]\right) \ldots = \dfrac{w}{8\ell^2}(\ell + a)^2(\ell - a)^2$

M_2 (at R_2) $\ldots = \dfrac{wa^2}{2}$

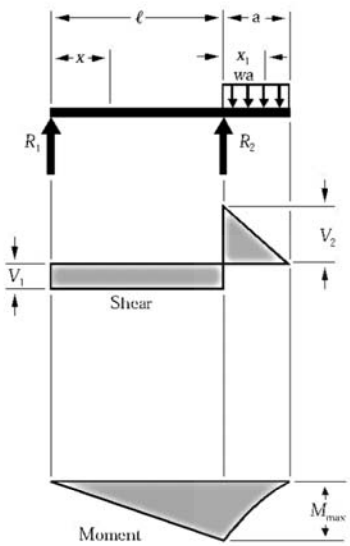
M_x (between supports) $\ldots = \dfrac{wx}{2\ell}(\ell^2 - a^2 - x\ell)$

M_{x_1} (for overhang) $\ldots = \dfrac{w}{2}(a - x_1)^2$

Δ_x (between supports) $\ldots = \dfrac{wx}{24EI\ell}(\ell^4 - 2\ell^2x^2 + \ell x^3 - 2a^2\ell^2 + 2a^2x^2)$

Δ_{x_1} (for overhang) $\ldots = \dfrac{wx_1}{24EI}(4a^2\ell - \ell^3 + 6a^2x_1 - 4ax_1^2 + x_1^3)$

Figure 19 **Beam Overhanging One Support – Uniformly Distributed Load on Overhang**



$R_1 = V_1 \dots = \frac{wa^2}{2\ell}$

$R_2 = V_1 + V_2 \dots = \frac{wa}{2\ell}(2\ell + a)$

$V_2 \dots = wa$

V_{x_1} (for overhang) $\dots = w(a - x_1)$

M_{max} (at R_2) $\dots = \frac{wa^2}{2}$

M_x (between supports) $\dots = \frac{wa^2 x}{2\ell}$

M_{x_1} (for overhang) $\dots = \frac{w}{2}(a - x_1)^2$

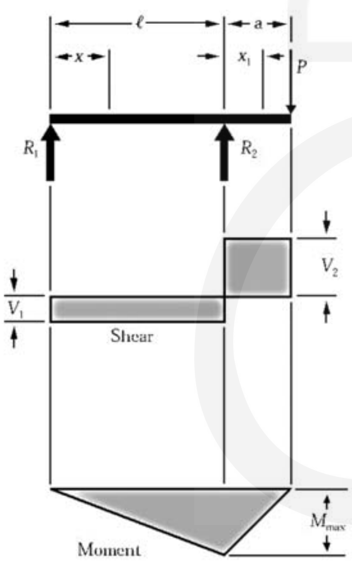
$\Delta_{max}\left(\text{between supports at } x = \frac{\ell}{\sqrt{3}}\right) = \frac{wa^2\ell^2}{18\sqrt{3}EI} = .03208\frac{wa^2\ell^2}{EI}$

Δ_{max} (for overhang at $x_1 = a$) $\dots = \frac{wa^3}{24EI}(4\ell + 3a)$

Δ_x (between supports) $\dots = \frac{wa^2 x}{12EI\ell}(\ell^2 - x^2)$

Δ_{x_1} (for overhang) $\dots = \frac{wx_1}{24EI}(4a^2\ell + 6a^2x_1 - 4ax_1^2 + x_1^3)$

Figure 20 **Beam Overhanging One Support – Concentrated Load at End of Overhang**



$R_1 = V_1 \dots = \frac{Pa}{\ell}$

$R_2 = V_1 + V_2 \dots = \frac{P}{\ell}(\ell + a)$

$V_2 \dots = P$

M_{max} (at R_2) $\dots = Pa$

M_x (between supports) $\dots = \frac{Pax}{\ell}$

M_{x_1} (for overhang) $\dots = P(a - x_1)$

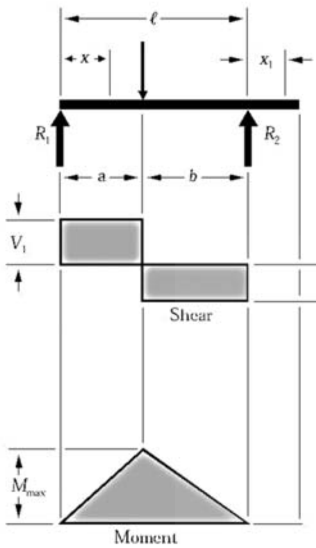
$\Delta_{max}\left(\text{between supports at } x = \frac{\ell}{\sqrt{3}}\right) = \frac{Pa\ell^2}{9\sqrt{3}EI} = .06415\frac{Pa\ell^2}{EI}$

Δ_{max} (for overhang at $x_1 = a$) $\dots = \frac{Pa^2}{3EI}(\ell + a)$

Δ_x (between supports) $\dots = \frac{Pax}{6EI\ell}(\ell^2 - x^2)$

Δ_{x_1} (for overhang) $\dots = \frac{Px_1}{6EI}(2a\ell + 3ax_1 - x_1^2)$

Figure 21 Beam Overhanging One Support – Concentrated Load at Any Point Between Supports



$R_1 = V_1$ (max when $a < b$) . . . . . . $= \dfrac{Pb}{\ell}$

$R_2 = V_2$ (max when $a > b$) . . . . . . $= \dfrac{Pa}{\ell}$

M_{max} (at point of load) . . . . $= \dfrac{Pab}{\ell}$

M_x (when $x < a$) . . . . $= \dfrac{Pbx}{\ell}$

$\Delta_{max}\left(\text{at } x = \sqrt{\dfrac{a(a+2b)}{3}} \text{ when } a > b\right)$. $= \dfrac{Pab(a+2b)\sqrt{3a(a+2b)}}{27EI\ell}$

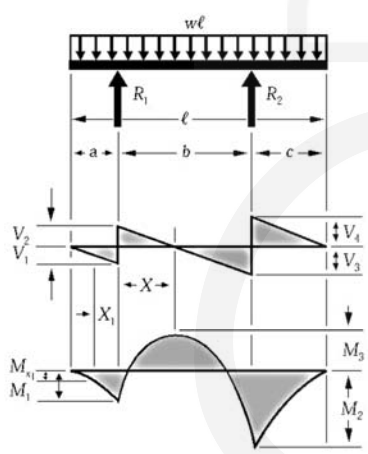
Δ_a (at point of load) . . . . $= \dfrac{Pa^2b^2}{3EI\ell}$

Δ_x (when $x < a$) . . . . $= \dfrac{Pbx}{6EI\ell}(\ell^2 - b^2 - x^2)$

Δ_x (when $x > a$) . . . . $= \dfrac{Pa(\ell - x)}{6EI\ell}(2\ell x - x^2 - a^2)$

Δ_{x_1} . . . . $= \dfrac{Pabx_1}{6EI\ell}(\ell + a)$

Figure 22 Beam Overhanging Both Supports – Unequal Overhangs – Uniformly Distributed Load



R_1 . . . . $= \dfrac{w\ell\,(\ell - 2c)}{2b}$

R_2 . . . . $= \dfrac{w\ell\,(\ell - 2a)}{2b}$

V_1 . . . . $= wa$

V_2 . . . . $= R_1 - V_1$

V_3 . . . . $= R_2 - V_4$

V_4 . . . . $= wc$

V_{x_1} . . . . $= V_1 - wx_1$

V_x (when $x < \ell$) . . . . $= R_1 - w(a + x_1)$

V_m (when $a < c$) . . . . $= R_2 - wc$

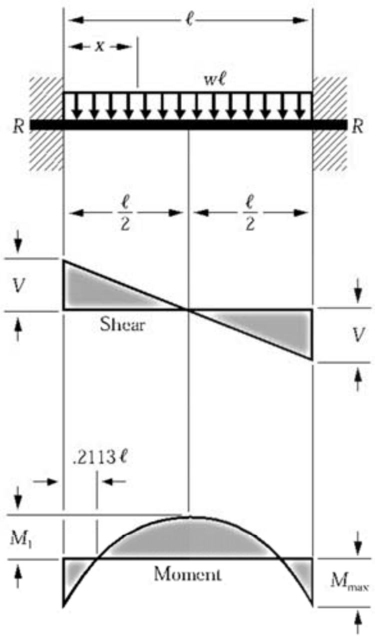
M_1 . . . . $= -\dfrac{wa^2}{2}$

M_2 . . . . $= -\dfrac{wc^2}{2}$

M_3 . . . . $= R_1\left(\dfrac{R_1}{2w} - a\right)$

$M_x\left(\text{max when } x = \dfrac{R_1}{w} - a\right)$ . . . . . $= R_1 x - \dfrac{w(a+x)^2}{2}$

Figure 23 Beam Fixed at Both Ends – Uniformly Distributed Load



$R = V \ldots\ldots = \frac{w\ell}{2}$

$V_x \ldots\ldots = w\left(\frac{\ell}{2} - x\right)$

M_{max} (at ends) $\ldots\ldots = \frac{w\ell^2}{12}$

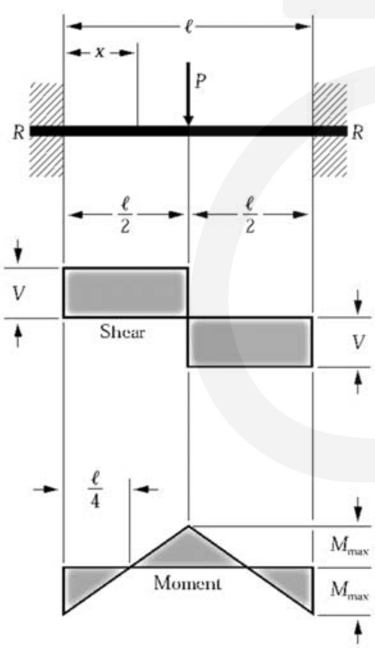
M_1 (at center) $\ldots\ldots = \frac{w\ell^2}{24}$

$M_x \ldots\ldots = \frac{w}{12}(6\ell x - \ell^2 - 6x^2)$

Δ_{max} (at center) $\ldots\ldots = \frac{w\ell^4}{384EI}$

$\Delta_x \ldots\ldots = \frac{wx^2}{24EI}(\ell - x)^2$

Figure 24 Beam Fixed at Both Ends – Concentrated Load at Center



$R = V \ldots\ldots = \frac{P}{2}$

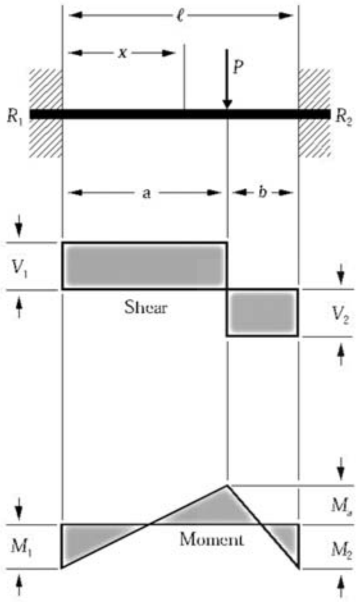
M_{max} (at center and ends) $\ldots\ldots = \frac{P\ell}{8}$

$M_x \left(\text{when } x < \frac{\ell}{2}\right) \ldots\ldots = \frac{P}{8}(4x - \ell)$

Δ_{max} (at center) $\ldots\ldots = \frac{P\ell^3}{192EI}$

$\Delta_x \left(\text{when } x < \frac{\ell}{2}\right) \ldots\ldots = \frac{Px^2}{48EI}(3\ell - 4x)$

Figure 25 Beam Fixed at Both Ends – Concentrated Load at Any Point



$R_1 = V_1$ (max when $a < b$) . . . . . . $= \dfrac{Pb^2}{\ell^3}(3a + b)$

$R_2 = V_2$ (max when $a > b$) . . . . . . $= \dfrac{Pa^2}{\ell^3}(a + 3b)$

M_1 (max when $a < b$) . . . . $= \dfrac{Pab^2}{\ell^2}$

M_2 (max when $a > b$) . . . . $= \dfrac{Pa^2b}{\ell^2}$

M_a (at point of load) . . . . $= \dfrac{2Pa^2b^2}{\ell^3}$

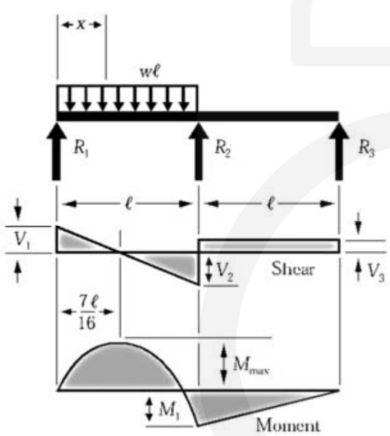
M_x (when $x < a$) . . . . $= R_1x - \dfrac{Pab^2}{\ell^2}$

$\Delta_{max}\left(\text{when } a > b \text{ at } x = \dfrac{2a\ell}{3a + b}\right)$ . . . $= \dfrac{2Pa^3b^2}{3EI(3a + b)^2}$

Δ_a (at point of load) . . . . $= \dfrac{Pa^3b^3}{3EI\ell^3}$

Δ_x (when $x < a$) . . . . $= \dfrac{Pb^2x^2}{6EI\ell^3}(3a\ell - 3ax - bx)$

Figure 26 Continuous Beam – Two Equal Spans – Uniform Load on One Span



$R_1 = V_1$ . . . . $= \dfrac{7}{16}w\ell$

$R_2 = V_2 + V_3$ . . . . $= \dfrac{5}{8}w\ell$

$R_3 = V_3$ . . . . $= -\dfrac{1}{16}w\ell$

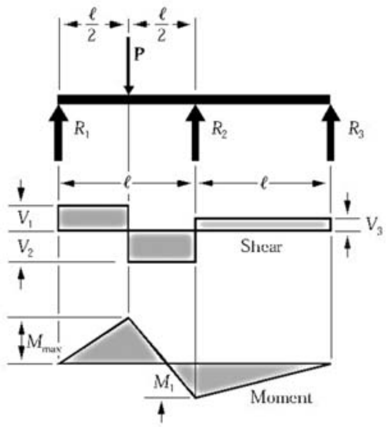
V_2 . . . . $= \dfrac{9}{16}w\ell$

$M_{max}\left(\text{at } x = \dfrac{7}{16}\ell\right)$ . . . . $= \dfrac{49}{512}w\ell^2$

M_1 (at support R_2) . . . . $= \dfrac{1}{16}w\ell^2$

M_x (when $x < \ell$) . . . . $= \dfrac{wx}{16}(7\ell - 8x)$

Figure 27 Continuous Beam – Two Equal Spans – Concentrated Load at Center of One Span



$R_1 = V_1 \dots = \frac{13}{32}P$

$R_2 = V_2 + V_3 \dots = \frac{11}{16}P$

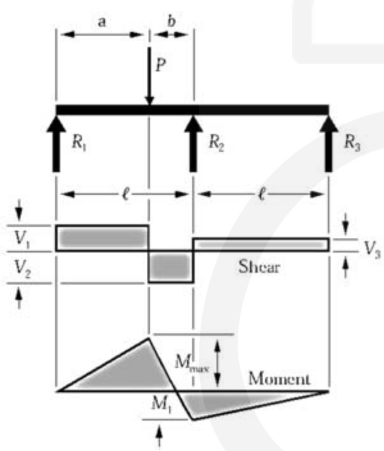
$R_3 = V_3 \dots = -\frac{3}{32}P$

$V_2 \dots = \frac{19}{32}P$

M_{max} (at point of load) $\dots = \frac{13}{64}P\ell$

M_1 (at support R_2) $\dots = \frac{3}{32}P\ell$

Figure 28 Continuous Beam – Two Equal Spans – Concentrated Load at Any Point



$R_1 = V_1 \dots = \frac{Pb}{4\ell^3}\left(4\ell^2 - a(\ell + a)\right)$

$R_2 = V_2 + V_3 \dots = \frac{Pa}{2\ell^3}\left(2\ell^2 + b(\ell + a)\right)$

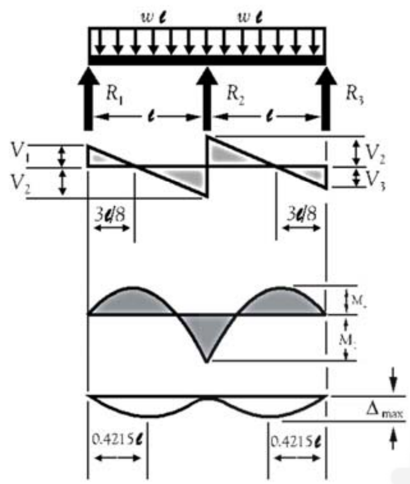
$R_3 = V_3 \dots = -\frac{Pab}{4\ell^3}(\ell + a)$

$V_2 \dots = \frac{Pa}{4\ell^3}\left(4\ell^2 + b(\ell + a)\right)$

M_{max} (at point of load) $\dots = \frac{Pab}{4\ell^3}\left(4\ell^2 - a(\ell + a)\right)$

M_1 (at support R_2) $\dots = \frac{Pab}{4\ell^2}(\ell + a)$

Figure 29 Continuous Beam – Two Equal Spans – Uniformly Distributed Load



$$R_1 = V_1 = R_3 = V_3 \; \ldots\ldots \; = \frac{3w\ell}{8}$$

$$R_2 \; \ldots\ldots \; = \frac{10w\ell}{8}$$

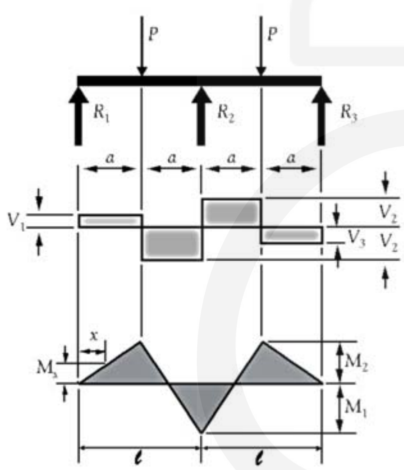
$$V_2 = V_{max} \; \ldots\ldots \; = \frac{5w\ell}{8}$$

$$M_1 \; \ldots\ldots \; = \frac{w\ell^2}{8}$$

$$M_2 \left(\text{at } \frac{3\ell}{8}\right) \; \ldots\ldots \; = \frac{9w\ell^2}{128}$$

$$\Delta_{max} \text{ (at } 0.4215\,\ell, \text{ approx. from } R_1 \text{ and } R_3) \; \ldots \; = \frac{w\ell^4}{185EI}$$

Figure 30 Continuous Beam – Two Equal Spans – Two Equal Concentrated Loads Symmetrically Placed



$$R_1 = V_1 = R_3 = V_3 \; \ldots\ldots \; = \frac{5P}{16}$$

$$R_2 = 2V_2 \; \ldots\ldots \; = \frac{11P}{8}$$

$$V_2 = P - R_1 \; \ldots\ldots \; = \frac{11P}{16}$$

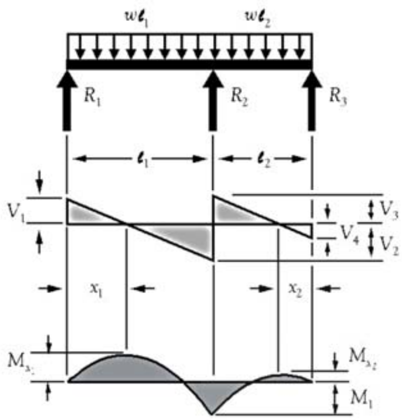
$$V_{max} \; \ldots\ldots \; = V_2$$

$$M_1 \; \ldots\ldots \; = -\frac{3P\ell}{16}$$

$$M_2 \; \ldots\ldots \; = \frac{5P\ell}{32}$$

$$M_x \text{ (when } x < a) \; \ldots\ldots \; = R_1 x$$

Figure 31 Continuous Beam – Two Unequal Spans – Uniformly Distributed Load



R_1 . . . . $= \dfrac{M_1}{\ell_1} + \dfrac{w\ell_1}{2}$

R_2 . . . . $= w\ell_1 + w\ell_2 - R_1 - R_3$

$R_3 = V_4$ . . . . $= \dfrac{M_1}{\ell_2} + \dfrac{w\ell_2}{2}$

V_1 . . . . $= R_1$

V_2 . . . . $= w\ell_1 - R_1$

V_3 . . . . $= w\ell_2 - R_3$

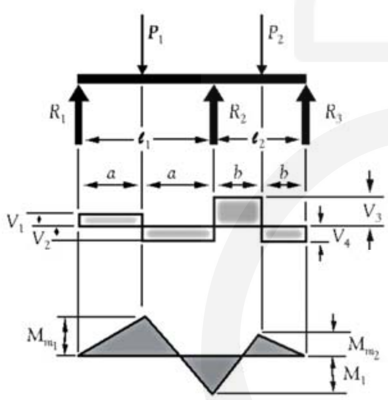
V_4 . . . . $= R_3$

M_1 . . . . $= -\dfrac{w\ell_2^{\,3} + w\ell_1^{\,3}}{8(\ell_1 + \ell_2)}$

$M_{x_1}\left(\text{when } x_1 = \dfrac{R_1}{w}\right)$ . . . . $= R_1x_1 - \dfrac{wx_1^2}{2}$

$M_{x_2}\left(\text{when } x_2 = \dfrac{R_3}{w}\right)$ . . . . $= R_3x_2 - \dfrac{wx_2^2}{2}$

Figure 32 Continuous Beam – Two Unequal Spans – Concentrated Load on Each Span Symmetrically Placed



R_1 . . . . $= \dfrac{M_1}{\ell_1} + \dfrac{P_1}{2}$

R_2 . . . . $= P_1 + P_2 - R_1 - R_3$

R_3 . . . . $= \dfrac{M_1}{\ell_2} + \dfrac{P_2}{2}$

V_1 . . . . $= R_1$

V_2 . . . . $= P_1 - R_1$

V_3 . . . . $= P_2 - R_3$

V_4 . . . . $= R_3$

M_1 . . . . $= -\dfrac{3}{16}\left(\dfrac{P_1\ell_1^{\,2} + P_2\ell_2^{\,2}}{\ell_1 + \ell_2}\right)$

M_{m_1} . . . . $= R_1a$

M_{m_2} . . . . $= R_3b$